Hints for the MATLAB Project
5 August 2005, jrm

Greetings All,

The MATLAB project was not supposed to be a killer math problem. The goal was to exercise your knowledge on a (hopefully) simple problem, and then let you develop your abilities with a modern computer tool, like MATLAB.

(Trust me on this, MATLAB is actually USED by folks in industry to solve all sorts of problems, I’ve seen it!)

I explained several approaches to this problem in class, but let me refresh your memories:

You could write differential equations for the circuit, then hand-solve these equations, then perform the transforms by hand, then use MATLAB to plot the results. This kind of minimizes the power that you can get out of using a computer tool.

Another approach is to write a differential equation, and then convert it to a difference equation, and then let MATLAB generate your h[n], pretend that is the h(t), and then have MATLAB generate the transforms and plot them.

Still another way: Model the circuit with Laplace transforms, and generate your H(s), and then generate your h(t). This is what I’ll show you here, but there are other approaches still. Get creative!

For the circuit given, you can produce the H(s) transfer function directly, using Voltage-division, with Laplace-transformed impedances:

\[
H(s) = \frac{R_1 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}}
\]

This can then be manipulated using simple algebra. I’ll show all of the steps for you:

\[
H(s) = \frac{R_1Cs + 1}{(R_1 + R_2)Cs + 1}
\]

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So now all you have to do is to make MATLAB show you the plots.

\[
H(s) = \left( \frac{R_1}{R_1 + R_2} \right) \left( \frac{s + \frac{1}{R_1 C} + \frac{1}{(R_1 + R_2)C} - \frac{1}{(R_1 + R_2)C}}{s + \frac{1}{(R_1 + R_2)C}} \right)
\]

\[
H(s) = \left( \frac{R_1}{R_1 + R_2} \right) \left( \frac{s + \frac{1}{(R_1 + R_2)C} + \frac{1}{R_1 C - \frac{1}{(R_1 + R_2)C}}}{s + \frac{1}{(R_1 + R_2)C}} \right)
\]

\[
H(s) = \left( \frac{R_1}{R_1 + R_2} \right) + \left( \frac{R_1}{R_1 + R_2} \right) \left( \frac{1}{R_1 C - \frac{1}{(R_1 + R_2)C}} \right)
\]

\[
H(s) = \left( \frac{R_1}{R_1 + R_2} \right) + \left[ \frac{1}{R_1 + R_2} \right] \left( \frac{1}{R_1 C - \frac{1}{(R_1 + R_2)C}} \right) \left( \frac{1}{s + \left( \frac{1}{(R_1 + R_2)C} \right)} \right)
\]

\[
H(s) = \left( \frac{R_1}{R_1 + R_2} \right) + \left[ \frac{R_2}{(R_1 + R_2)^2 C} \right] \left( \frac{1}{s + \left( \frac{1}{(R_1 + R_2)C} \right)} \right)
\]

Giving an \( h(t) \):

\[
h(t) = \left( \frac{R_1}{R_1 + R_2} \right) \delta(t) + \left[ \frac{R_2}{(R_1 + R_2)^2 C} \right] e^{-\frac{1}{(R_1 + R_2)C}} u(t)
\]

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